Decimal Floating-Point: Algorism for Computers

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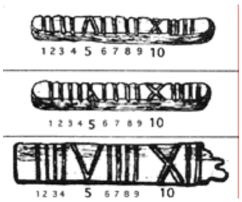
decArith16

Overview

- Why decimal arithmetic is increasingly important
- Why hardware support is needed
- How decimal arithmetic is done

Origins of decimal arithmetic

- Decimal (base 10) arithmetic has been used for thousands of years
- Algorism (Indo-Arabic place value system) in use since 800 AD
- Many early computers were decimal



Decimal computation today

- Pervasive in financial, commercial, and human-centric applications
  - often a legal requirement
- Benchmarks do not reflect actual use
- 55% of numeric data in databases are decimal (and a further 43% integers)

Why floating-point?

- Traditional integer arithmetic with 'manual' scaling is awkward and error-prone
- Floating-point is increasingly necessary
  - division and exponentiation
  - interest calculated daily
  - telephone calls priced by the second
  - calculators, financial analysis, *etc.*

Why not use binary FP?

- binary fractions *cannot* exactly represent all decimal fractions
- 1.2 x 1.2 ➔ 1.44 ?
  - 1.2 in a 32-bit binary float is actually: 1.2000000476837158203125
  - and this squared is: 1.44000005722045898437 5

A financial example…

- 5% sales tax on a $ 0.70 telephone call, rounded to the nearest cent
- 1.05 x 0.70 using binary double type is

  0.73499999999999998667732370449812151491641998291015625

  (should have been 0.735)
- rounds to $ 0.73, instead of $ 0.74

Hence…

- Binary floating-point cannot be used for commercial applications
  - cannot match values computed by hand
  - cannot meet legal and financial requirements, which are based on 2,500+ years of decimal arithmetic
- So applications use decimal software floating-point packages…

...but decimal software is slow…

- some measurements …

times in μs	x=y+z	x=y*z	x=y/z
Java BigDecimal	1.250	1.100	8.440
Binary hardware	0.006	0.006	0.078
Decimal penalty	208x	183x	108x

(These are 9-digit timings.   27-digit calculations are 9x worse for multiply and divide.)

Effect on real applications

- The 'telco' billing application
  - 1,000,000 calls read from file, priced, taxed, and printed (two minutes-worth)

	fastest C package	Java BigDecimal
time in decimal operations	72.2%	93.2%

The path to hardware…

- A 2x to 10x performance improvement in applications makes hardware support very attractive
- IEEE 854 tells us how to compute the *value* of floating-point results
- We can use redundant encodings to allow fixed-point and integer arithmetic, too

Traditional two-integer form

- Allows integer, fixed-point, and floating-point numbers in one representation
  - integers always have exponent = 0
  - in general: *numbers with the same number of decimal places have the same exponent, and need no alignment for addition*

e.g., 1.23 and 123.45 both have exponent -2

[123, -2] and [12345, -2]

Example: multiplication

- The significands are multiplied (an integer operation), and the exponent is the sum of the operand exponents

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123E-2 x 45E-1 gives 5535E-3
122E-2 x 45E-1 gives 5490E-3
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- Independent calculations for the two parts
- No further processing is necessary unless rounding (*etc.*) is needed

Rounding

- Correct rounding, as in IEEE 754/854
  - additional rounding mode (round-half-up)
- A rounded normal number will always have maximum digits (the first digit will be non-zero)
- Subnormals may have leading zero digit(s)

Note …

- The core operations when no rounding occurs are simple integer operations; integer arithmetic is a subset
- Comparison does not distinguish between redundant encodings of the same value
- The rules are base-independent

Integer-based floating-point

- Compatible with:
  - IEEE 754/854
  - manual processes (algorism)
  - legal requirements
  - programming language data types (COBOL, PL/I, Java, C#, Rexx, Visual Basic, *etc.*)
  - databases (DB2, SQL Server, Oracle, *etc.*)
  - application testcase data formats
  - mixed-type arithmetic: 12 x $ 9.99

Summary

- Hardware two-integer arithmetic (or the equivalent) gives same results as software
  - allows the hardware to be used to accelerate existing applications and processes
  - *e.g.,* a typical large bank has 1,480 programmers, 65 application subsystems, 900+ IT projects/year, and 10,000+ programs in use
- This does not conflict with IEEE 754/854
  - allows integer and FP in the same unit

Questions?

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(Google: decimal arithmetic)